GLOBAL EDITION

# OPTIONS, FUTURES, AND OTHER DERIVATIVES

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**To Michelle**

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# PREFACE

Derivatives markets have seen many changes over the last 30 years. Successive editions of *Options, Futures, and Other Derivatives* have managed to keep up to date. The book has an applied approach. It is a very popular college text, but it can also be found on trading-room desks throughout the world. (Indeed, I receive emails from derivatives practitioners about the book almost every day.) The blending of material useful for practitioners with material appropriate for university courses is what makes the book unique.

## NEW TO THIS EDITION

- A major change in financial markets will be the phase-out of LIBOR. This has led to important changes throughout the 11th edition. The overnight reference rates that will replace LIBOR, and the way they are used to determine zero curves, are discussed carefully.
- Within-chapter examples and end-of-chapter problems that were previously based on LIBOR have been largely replaced by examples based on the new reference rates or by generic examples.
- The likely impact of the new reference rates on valuation models is discussed.
- The new reference rates are considered to be risk-free whereas LIBOR incorporates a time-varying credit spread. The book discusses the desire on the part of banks to augment the new reference rates with a measure of the level of credit spreads in the market.
- The chapter on Wiener processes now covers fractional Brownian motion. This is becoming increasingly used in modeling volatility.
- Rough volatility models which have in the last few years been found to fit volatility surfaces well are added to the models considered in Chapter 27.
- Machine learning is becoming increasingly used in pricing and hedging derivatives. The reader is introduced to these applications at various points in the book.
- Changes in the regulatory environment, including Basel IV, are covered.
- The end-of-chapter problems have been updated. To make the book as easy to use as possible, solutions to all end-of-chapter problems are now on www.pearsonglobaleditions.com and www-2.rotman.utoronto.ca/~hull.

- Instructor support material has been revised. In particular, there are now many more suggestions on assignment questions that can be used in conjunction with chapters.
- The DerivaGem software is less LIBOR-focused and is available for download from www-2.rotman.utoronto.ca/~hull/software.
- Tables, charts, market data, and examples have been updated throughout the book.

## SOLVING TEACHING AND LEARNING CHALLENGES

Most instructors find that courses in derivatives are fun to teach. There is not a big gap between theory and practice. Most students know a little about the subject and are motivated to learn more. Usually there is some current news that can be discussed in class, e.g., the level of the VIX index or events that affect particular option prices.

### *Math Knowledge*

Math is the key challenge for many students taking a course in derivatives. I have kept this in mind in the way material is presented throughout the book. Instructors are often faced with a trade-off between mathematical rigor and the simplicity with which an idea is explained. My preference is always to look for the simplest way of explaining an idea in the first instance. Sometimes using words rather than equations is effective. I avoid using notation that has lots of subscripts, superscripts, and function arguments as far as possible because this can be off-putting to a reader who is new to the material. Nonessential mathematical material has been either eliminated or included in technical notes on my website.

The reality is that many students only understand an equation when they have seen numbers substituted into it. For that reason, many numerical examples have been included in the text. The software DerivaGem (discussed below) allows students to get a feel for equations by trying different inputs.

I am often asked about the math prerequisites for *Options, Futures and Other Derivatives*. Students will be able to cope with a course based on this book if they are comfortable with algebra and understand probabilities and probability distributions. A knowledge of calculus concepts is useful for parts of the book. But no knowledge of stochastic calculus is assumed. The basic knowledge of stochastic processes that is needed for a more advanced understanding of derivatives is explained carefully in Chapter 14.

### *End of Chapter Problems*

As in earlier editions, there are many other end-of-chapter problems to help students apply the ideas presented in the chapters. These have been updated. The distinction between “practice questions” and “further questions” has been eliminated. Answers to all end-of-chapter problems are on my website and available through www.pearsonglobaleditions.com.

## *Designing a Course*

There are many ways in which the material in the book can be used. Instructors teaching an introductory course in derivatives tend to spend most time on the first 20 chapters, and often choose to omit Chapter 14 and Section 15.6. Instructors teaching a more advanced course find that many different combinations of chapters in the second half of the book can be used. I find that Chapter 37 is a fun chapter that works well at the end of either an introductory or an advanced course.

## *Software*

The DerivaGem software is an important part of the book. Students get comfortable with the models presented in the book when they use DerivaGem to value transactions under different assumptions. The use of the software is explained at the end of the book.

I recommend giving students assignments that involve using the basic DG400a.xls software. There are many types of assignments that can be developed. For example, students can be asked to compare American or European option prices given by a binomial model with those from the Black–Scholes–Merton model. They can be asked to report what happens as the number of time steps is increased in a binomial model and can use the software to display trees. (DerivaGem can display trees with up to 10 time steps and can calculate prices and Greek letters using up to 500 time steps.)

Many charts can be produced using the software and students can include those charts in reports produced for the instructor. The calculation of zero curves and swap valuation is made easy with DerivaGem. I like to use DerivaGem in class when I illustrate some key concepts.

Students taking a more advanced course in derivatives can be asked to compare prices given by different models using the *Alternative Models* worksheet in DG400a.xls. Alternatives to Black–Scholes that are covered include CEV, Merton mixed jump–diffusion, variance gamma, Heston, and SABR. Students can also be asked to carry out assignments concerned with the use of different models for pricing bond options. The CDS and CDO worksheets can be used in conjunction with each other for an assignment if CDOs are covered.

DerivaGem can be used in conjunction with current market data that can be downloaded from Yahoo Finance or other providers. For example, students can be asked to compare implied volatilities for options on different stocks that have been in the news. They can also be asked to calculate volatility term structures and volatility smiles for stock indices. Assignments such as these can be important because they make the underlying concepts more "real" and lead to interesting classroom discussions.

The *DG400 Applications* software enables students to carry out assignments where they are asked investigate issues such as how the performance of delta hedging is improved as the interval between rebalancing is decreased or how managing gamma can improve the performance of delta hedging. Assuming students have a basic knowledge of Excel, they should have no difficulty using this software and changing instructions as necessary.

The *DG400 Functions* software is a little more challenging. It contains the functions used by DG400a.xls. Students can use these functions to develop their own Excel worksheets in order to investigate particular issues and answer assignment questions.

Many instructors find DerivaGem to be a really useful resource. DerivaGem can be downloaded from www-2.rotman.utoronto.ca/~hull/software.

### *Slides*

Several hundred PowerPoint slides accompany this book. They can be a useful starting point for instructors. Those who adopt the text are welcome to adapt the slides to meet their needs. These slides are available on www.pearsonglobaleditions.com and www-2.rotman.utoronto.ca/~hull.

### *Technical Notes*

There are over 30 technical notes available. They are referred to in the text and can be downloaded from www-2.rotman.utoronto.ca/~hull/TechnicalNotes.

By not including the Technical Notes in the book, I am able to streamline the presentation of material so that it is more reader-friendly.

## EMPLOYABILITY

A natural question for students is: "Will a course in derivatives improve my chances of a getting a job in finance?" The answer is an overwhelming yes. Probably the first thing many students think about when considering options or other derivatives is an exchange such as the CBOE. In fact, as Chapter 1 makes clear, the over-the-counter (OTC) market is much larger than the exchange-traded market and likely to be much more important to students in their first job (or subsequent jobs). *Options, Futures, and Other Derivatives* has a much bigger focus on the OTC market than most other derivatives texts.

Derivatives have steadily increased in importance. Potential employers can be classified as "buy side" and "sell side". The buy side includes nonfinancial corporations, insurance companies, fund managers, and some other financial institutions. The sell side consists of large financial institutions who act as market makers. Many students who take courses in derivatives may not become derivatives traders or derivatives analysts. However, derivatives now permeate all aspects of finance. If you work in investment banking, there is likely to be a derivatives component to some of the deals you are involved in; if you work in fund management, you will probably find derivatives to be convenient tools for some purposes; if you work for a nonfinancial corporation, you may be involved in using derivative contracts for hedging and negotiating with a sell-side institution; and so on. Whatever your role in finance, it is important that you be able to talk about derivatives knowledgeably, use the right words, and understand the motivations of a counterparty to a transaction. A course based on *Options, Futures, and Other Derivatives* will help you do this.

What about those of you who want to specialize in derivatives? I have literally lost count of the many successful derivative executives who have told me "Thank you for your book. I read it before the interview, and it got me my first job in derivatives." (My joking response has typically been: "Great, but you realize that means you owe me 20% of your first year's salary.") The people I am talking about typically had engineering, physics, or other quantitative backgrounds at the time of the interview but had never taken a course in finance! So, while the book is important for those planning a career in finance, it is absolutely essential reading for all those aspiring to a career in derivatives. As mentioned earlier, it is found on trading-room desks throughout the world.

This book will help you develop your quant skills so that you become more marketable in finance. But other skills are of course important. Good communication skills are necessary. Many instructors ask students to present the results of projects in class. Students should take full advantage of these opportunities to practice and improve. If presentations are recorded, they should review the recording carefully.

At my business school, we used to run optional mock interviews and other self-development activities for students. Interestingly, the students that took advantage of them tended to be the ones that already had fairly good skills. The students that really needed help did not participate. (We have since made the activities mandatory.) I would urge all students to take advantage of all opportunities to improve their soft skills. Do not dismiss them as unimportant.

What are other important skills? The book discusses the regulatory environment for derivatives which changed a lot following the 2008 financial crisis. Make sure you understand the issues and are familiar with the latest developments. You should also use a derivatives course to help develop your critical thinking skills. Ask questions in class and do not be afraid to express an opinion about an issue.

A potential employer will want to be convinced that you can work well with others. While at university you will be involved in many group projects and should take this opportunity to develop good collaboration skills. You may find some members of your group difficult to work with, but this is also likely to be true in your first full-time job. Go to an interview prepared to talk about your experiences working with other students.

In addition to quant skills and knowledge of derivatives, I have mentioned that communication skills, the ability to work collaboratively, and critical thinking are soft skills that you should try and develop to make sure you get that first job. Another I might add is social responsibility. It is not an accident that most successful corporate executives are actively involved in community activities. Be prepared to talk about sustainable finance, which is an aspect of social responsibility and becoming an increasingly important area within finance.

## ACKNOWLEDGMENTS

Many people have played a part in the development of successive editions of this book. Indeed, the list of people who have provided me with feedback on the book is now so long that it is not possible to mention everyone. I have benefited from the advice of many academics who have taught from the book and from the comments of many derivatives practitioners. I would like to thank the students in my courses at the University of Toronto who have made many suggestions on how the material can be improved. Eddie Mizzi from The Geometric Press did an excellent job editing the final manuscript and handling page composition. Emilio Barone from Luiss Guido Carli University in Rome provided many detailed comments. Andrés Olivé provided valuable research assistance.

Alan White, a colleague at the University of Toronto, deserves a special acknowledgment. Alan and I have been carrying out joint research and consulting in the areas of derivatives and risk management for over 30 years. During that time, we have spent many hours discussing key issues. Many of the new ideas in this book, and many of the new ways used to explain old ideas, are as much Alan's as mine. Alan has done most of the development work on the DerivaGem software.

Special thanks are due to the many people at Pearson I have worked with for over 30 years. Those who have worked with me on the 11th edition include Neeraj Bhalla, Sugandh Juneja, and Emily Biberger. I would like to thank them for their enthusiasm, advice, and encouragement.

I welcome comments on the book from readers. My e-mail address is:

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John Hull

## About the Author

John Hull is the Maple Financial Professor of Derivatives and Risk Management at the Joseph L. Rotman School of Management, University of Toronto. He was in 2016 awarded the title of University Professor (an honor granted to only 2% of faculty at the University of Toronto). He is an internationally recognized authority on derivatives and risk management and has many publications in this area. His work has an applied focus. He has acted as consultant to many financial institutions throughout the world and has won many teaching awards, including the University of Toronto's prestigious Northrop Frye award. His research and teaching activities include risk management, regulation, and machine learning, as well as derivatives. He is co-director of Rotman's Master of Finance and Master of Financial Risk Management programs.

# CHAPTER 1 Introduction

In the last 40 years, derivatives have become increasingly important in finance. Futures and options are actively traded on many exchanges throughout the world. Many different types of forward contracts, swaps, options, and other derivatives are entered into by financial institutions, fund managers, and corporate treasurers in the over-the-counter market. Derivatives are added to bond issues, used in executive compensation plans, embedded in capital investment opportunities, used to transfer risks in mortgages from the original lenders to investors, and so on. We have now reached the stage where those who work in finance, and many who work outside finance, need to understand how derivatives work, how they are used, and how they are priced.

Whether you love derivatives or hate them, you cannot ignore them! The derivatives market is huge—much bigger than the stock market when measured in terms of underlying assets. The value of the assets underlying outstanding derivatives transactions is several times the world gross domestic product. As we shall see in this chapter, derivatives can be used for hedging or speculation or arbitrage. They can transfer a wide range of risks in the economy from one entity to another.

A *derivative* involves two parties agreeing to a future tranasaction. Its value depends on (or derives from) the values of other underlying variables. Very often the variables underlying derivatives are the prices of traded assets. A stock option, for example, is a derivative whose value is dependent on the price of a stock. However, derivatives can be dependent on almost any variable, from the price of hogs to the amount of snow falling at a certain ski resort.

Since the first edition of this book was published in 1988 there have been many developments in derivatives markets. For example:

- Many new instruments such as credit derivatives, electricity derivatives, weather derivatives, and insurance derivatives have been developed.
- Many new types of interest rate, foreign exchange, and equity derivatives now trade.
- There have been many new ideas in risk management and risk measurement.
- Real option methods for capital investment appraisal have been developed.
- The financial crisis of 2008 occurred, with derivatives (perhaps unfairly) getting much of the blame.

- Many regulations affecting the over-the-counter derivatives market have been introduced.
- The “risk-free” discount rate used to value derivatives has changed and the decision has been taken to phase out LIBOR.
- Derivatives dealers now adjust the way they price derivatives to allow for credit risks, funding costs, and capital requirements.
- Collateral and credit issues are now given much more attention and have led to changes in the way derivatives are traded.
- Machine learning is now becoming widely used for managing derivatives portfolios.

The book has evolved to keep up to date with these developments. For example: the 2008 financial crisis is discussed in Chapter 8; changes in the interest rates used for derivatives pricing are discussed in Chapter 4; valuation adjustments are covered in Chapter 9; real options are explained in Chapter 36; credit derivatives are covered in Chapter 25; energy, weather, and insurance derivatives are covered in Chapter 35. Machine learning applications are discussed at various points in the book.

In this opening chapter, we take a first look at derivatives markets and how they are changing. We contrast exchange-traded and over-the-counter derivatives markets and review recent regulatory changes affecting the markets. We describe forward, futures, and options markets and provide examples of how they are used by hedgers, speculators, and arbitrageurs. Later in the book we will elaborate on many of the points made in this chapter.

## 1.1 EXCHANGE-TRADED MARKETS

A derivatives exchange is a market where individuals and companies trade standardized contracts that have been defined by the exchange. Derivatives exchanges have existed for a long time. The Chicago Board of Trade (CBOT) was established in 1848 to bring farmers and merchants together. Initially its main task was to standardize the quantities and qualities of the grains that were traded. Within a few years, the first futures-type contract was developed. It was known as a *to-arrive contract*. Speculators soon became interested in the contract and found trading the contract to be an attractive alternative to trading the grain itself. A rival futures exchange, the Chicago Mercantile Exchange (CME), was established in 1919. Now futures exchanges exist all over the world. (See table at the end of the book.) The CME and CBOT have merged to form the CME Group (www.cmegroup.com), which also includes the New York Mercantile Exchange (NYMEX), and the Kansas City Board of Trade (KCBT).

The Chicago Board Options Exchange (CBOE, www.cboe.com) started trading call option contracts on 16 stocks in 1973. Options had traded prior to 1973, but the CBOE succeeded in creating an orderly market with well-defined contracts. Put option contracts started trading on the exchange in 1977. The CBOE now trades options on thousands of stocks and many different stock indices. Like futures, options have proved to be very popular contracts. Many other exchanges throughout the world now trade options. (See table at the end of the book.) The underlying assets include foreign currencies and futures contracts as well as stocks and stock indices.

Once two traders have agreed to trade a product offered by an exchange, it is handled by the exchange clearing house. This stands between the two traders and manages the risks. Suppose, for example, that trader A enters into a futures contract to buy 100 ounces of gold from trader B in six months for $1,750 per ounce. The result of this trade will be that A has a contract to buy 100 ounces of gold from the clearing house at $1,750 per ounce in six months and B has a contract to sell 100 ounces of gold to the clearing house for $1,750 per ounce in six months. The advantage of this arrangement is that traders do not have to worry about the creditworthiness of the people they are trading with. The clearing house takes care of credit risk by requiring each of the two traders to deposit funds (known as margin) with the clearing house to ensure that they will live up to their obligations. Margin requirements and the operation of clearing houses are discussed in more detail in Chapter 2.

### Electronic Markets

Traditionally derivatives exchanges have used what is known as the *open outcry system*. This involves traders physically meeting on the floor of the exchange, shouting, and using a complicated set of hand signals to indicate the trades they would like to carry out. Exchanges have largely replaced the open outcry system by *electronic trading*. This involves traders entering their desired trades at a keyboard and a computer being used to match buyers and sellers. The open outcry system has its advocates, but, as time passes, it is becoming less and less used.

Electronic trading has led to a growth in high-frequency trading. This involves the use of algorithms to initiate trades, often without human intervention, and has become an important feature of derivatives markets.

## 1.2 OVER-THE-COUNTER MARKETS

Not all derivatives trading is on exchanges. Many trades take place in the *over-the-counter* (OTC) market. Banks, other large financial institutions, fund managers, and corporations are the main participants in OTC derivatives markets. Once an OTC trade has been agreed, the two parties can either present it to a central counterparty (CCP) or clear the trade bilaterally. A CCP is like an exchange clearing house. It stands between the two parties to the derivatives transaction so that one party does not have to bear the risk that the other party will default. When trades are cleared bilaterally, the two parties have usually signed an agreement covering all their transactions with each other. The issues covered in the agreement include the circumstances under which outstanding transactions can be terminated, how settlement amounts are calculated in the event of a termination, and how the collateral (if any) that must be posted by each side is calculated. CCPs and bilateral clearing are discussed in more detail in Chapter 2.

Large banks often act as market makers for the more commonly traded instruments. This means that they are always prepared to quote a bid price (at which they are prepared to take one side of a derivatives transaction) and an ask price (at which they are prepared to take the other side).

**Business Snapshot 1.1** The Lehman Bankruptcy

On September 15, 2008, Lehman Brothers filed for bankruptcy. This was the largest bankruptcy in U.S. history and its ramifications were felt throughout derivatives markets. Almost until the end, it seemed as though there was a good chance that Lehman would survive. A number of companies (e.g., the Korean Development Bank, Barclays Bank in the United Kingdom, and Bank of America) expressed interest in buying it, but none of these was able to close a deal. Many people thought that Lehman was "too big to fail" and that the U.S. government would have to bail it out if no purchaser could be found. This proved not to be the case.

How did this happen? It was a combination of high leverage, risky investments, and liquidity problems. Commercial banks that take deposits are subject to regulations on the amount of capital they must keep. Lehman was an investment bank and not subject to these regulations. By 2007, its leverage ratio had increased to 31:1, which means that a 3–4% decline in the value of its assets would wipe out its capital. Dick Fuld, Lehman's Chairman and Chief Executive Officer, encouraged an aggressive deal-making, risk-taking culture. He is reported to have told his executives: "Every day is a battle. You have to kill the enemy." The Chief Risk Officer at Lehman was competent, but did not have much influence and was even removed from the executive committee in 2007. The risks taken by Lehman included large positions in the instruments created from subprime mortgages, which will be described in Chapter 8. Lehman funded much of its operations with short-term debt. When there was a loss of confidence in the company, lenders refused to renew this funding, forcing it into bankruptcy.

Lehman was very active in the over-the-counter derivatives markets. It had over a million transactions outstanding with about 8,000 different counterparties. Lehman's counterparties were often required to post collateral and this collateral had in many cases been used by Lehman for various purposes. Litigation aimed at determining who owes what to whom continued for many years after the bankruptcy filing.

Prior to the financial crisis, which started in 2007 and is discussed in some detail in Chapter 8, OTC derivatives markets were largely unregulated. Following the financial crisis and the failure of Lehman Brothers (see Business Snapshot 1.1), we have seen the development of many new regulations affecting the operation of OTC markets. The main objectives of the regulations are to improve the transparency of OTC markets and reduce systemic risk (see Business Snapshot 1.2). The over-the-counter market in some respects is being forced to become more like the exchange-traded market. Three important changes are:

1. Standardized OTC derivatives between two financial institutions in the United States must, whenever possible, be traded on what are referred to a *swap execution facilities* (SEFs). These are platforms similar to exchanges where market participants can post bid and ask quotes and where market participants can trade by accepting the quotes of other market participants.
2. There is a requirement in most parts of the world that a CCP be used for most standardized derivatives transactions between financial institutions.
3. All trades must be reported to a central repository.

**Business Snapshot 1.2** Systemic Risk

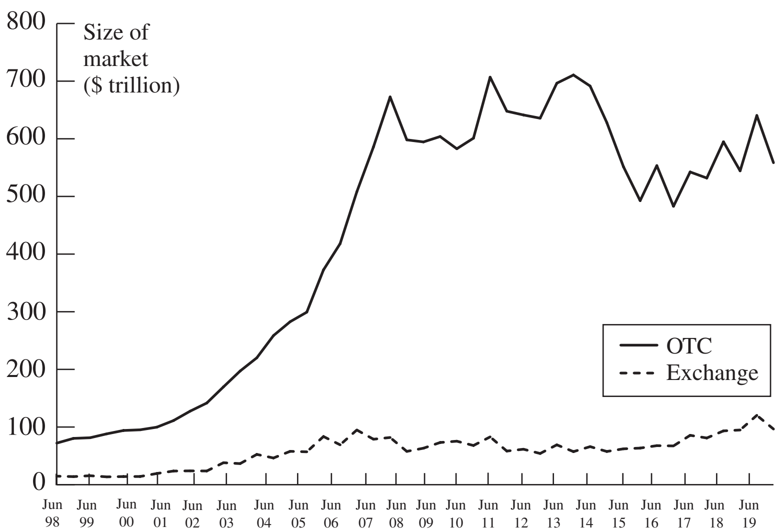
Systemic risk is the risk that a default by one financial institution will create a "ripple effect" that leads to defaults by other financial institutions and threatens the stability of the financial system. There are huge numbers of over-the-counter transactions between banks. If Bank A fails, Bank B may take a huge loss on the transactions it has with Bank A. This in turn could lead to Bank B failing. Bank C that has many outstanding transactions with both Bank A and Bank B might then take a large loss and experience severe financial difficulties; and so on.

The financial system has survived defaults such as Drexel in 1990 and Lehman Brothers in 2008, but regulators continue to be concerned. During the market turmoil of 2007 and 2008, many large financial institutions were bailed out, rather than being allowed to fail, because governments were concerned about systemic risk.

## Market Size

Both the over-the-counter and the exchange-traded market for derivatives are huge. The number of derivatives transactions per year in OTC markets is smaller than in exchange-traded markets, but the average size of the transactions is much greater. Although the statistics that are collected for the two markets are not exactly comparable, it is clear that the volume of business in the over-the-counter market is much larger than in the exchange-traded market. The Bank for International Settlements (www.bis.org) started collecting statistics on the markets in 1998. Figure 1.1 compares (a) the estimated total principal amounts underlying transactions that were outstanding in the over-the-counter markets between June 1998 and December 2019 and (b) the estimated total value of the assets underlying exchange-traded contracts during the same period. Using these measures, the size of the over-the-counter market in December 2019 was $558.5 trillion

**Figure 1.1** Size of over-the-counter and exchange-traded derivatives markets.



and the size of the exchange-traded market was \$96.5 trillion.[1] Figure 1.1 shows that the OTC market grew rapidly up to 2007, but has seen very little net growth since then. One reason for the lack of growth is the popularity of *compression*. This is a procedure where two or more counterparties restructure transactions with each other with the result that the underlying principal is reduced.

In interpreting Figure 1.1, we should bear in mind that the principal underlying an over-the-counter transaction is not the same as its value. An example of an over-the-counter transaction is an agreement to buy 100 million U.S. dollars with British pounds at a predetermined exchange rate in 1 year. The total principal amount underlying this transaction is \$100 million. However, the value of the transaction might be only \$1 million. The Bank for International Settlements estimates the gross market value of all over-the-counter transactions outstanding in December 2019 to be about \$11.6 trillion.[2]

## 1.3 FORWARD CONTRACTS

A relatively simple derivative is a *forward contract*. It is an agreement to buy or sell an asset at a certain future time for a certain price. It can be contrasted with a *spot contract*, which is an agreement to buy or sell an asset almost immediately. A forward contract is traded in the over-the-counter market—usually between two financial institutions or between a financial institution and one of its clients.

One of the parties to a forward contract assumes a *long position* and agrees to buy the underlying asset on a certain specified future date for a certain specified price. The other party assumes a *short position* and agrees to sell the asset on the same date for the same price.

Forward contracts on foreign exchange are very popular. Most large banks employ both spot and forward foreign-exchange traders. As we shall see in Chapter 5, there is a relationship between forward prices, spot prices, and interest rates in the two currencies. Table 1.1 provides quotes for the exchange rate between the British pound (GBP) and the U.S. dollar (USD) that might be made by a large international bank on May 21, 2020. The quote is for the number of USD per GBP. The first row indicates that the

**Table 1.1** Spot and forward quotes for the exchange rate between USD and GBP on May 21, 2020 (GBP = British pound; USD = U.S. dollar; quote is number of USD per GBP).

| | *Bid* | *Ask* |
|---|---|---|
| Spot | 1.2217 | 1.2220 |
| 1-month forward | 1.2218 | 1.2222 |
| 3-month forward | 1.2220 | 1.2225 |
| 6-month forward | 1.2224 | 1.2230 |

[1] When a CCP stands between two sides in an OTC transaction, two transactions are considered to have been created for the purposes of the BIS statistics.

[2] A contract that is worth \$1 million to one side and −\$1 million to the other side would be counted as having a gross market value of \$1 million.

bank is prepared to buy GBP (also known as sterling) in the spot market (i.e., for virtually immediate delivery) at the rate of \$1.2217 per GBP and sell sterling in the spot market at \$1.2220 per GBP. The second, third, and fourth rows indicate that the bank is prepared to buy sterling in 1, 3, and 6 months at \$1.2218, \$1.2220, and \$1.2224 per GBP, respectively, and to sell sterling in 1, 3, and 6 months at \$1.2222, \$1.2225, and \$1.2230 per GBP, respectively.

Forward contracts can be used to hedge foreign currency risk. Suppose that, on May 21, 2020, the treasurer of a U.S. corporation knows that the corporation will pay £1 million in 6 months (i.e., on November 21, 2020) and wants to hedge against exchange rate moves. Using the quotes in Table 1.1, the treasurer can agree to buy £1 million 6 months forward at an exchange rate of 1.2230. The corporation then has a long forward contract on GBP. It has agreed that on November 21, 2020, it will buy £1 million from the bank for \$1.2230 million. The bank has a short forward contract on GBP. It has agreed that on November 21, 2020, it will sell £1 million for \$1.2230 million. Both sides have made a binding commitment.

## Payoffs from Forward Contracts

Consider the position of the corporation in the trade we have just described. What are the possible outcomes? The forward contract obligates the corporation to buy £1 million for \$1,223,000. If the spot exchange rate rose to, say, 1.3000, at the end of the 6 months, the forward contract would be worth \$77,000 (= \$1,300,000 − \$1,223,000) to the corporation. It would enable £1 million to be purchased at an exchange rate of 1.2230 rather than 1.3000. Similarly, if the spot exchange rate fell to 1.2000 at the end of the 6 months, the forward contract would have a negative value to the corporation of \$23,000 because it would lead to the corporation paying \$23,000 more than the market price for the sterling.

In general, the payoff from a long position in a forward contract on one unit of an asset is

$$S_T - K$$

where $K$ is the delivery price and $S_T$ is the spot price of the asset at maturity of the contract. This is because the holder of the contract is obligated to buy an asset worth $S_T$ for $K$. Similarly, the payoff from a short position in a forward contract on one unit of an asset is

$$K - S_T$$

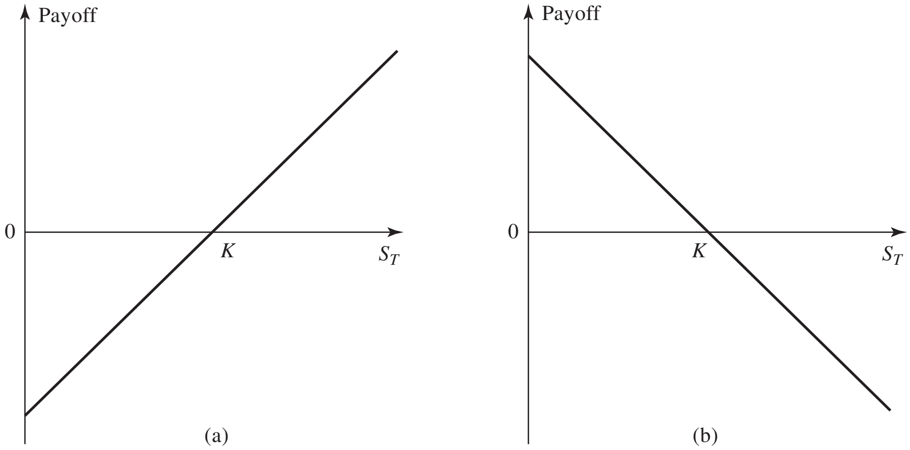
These payoffs can be positive or negative. They are illustrated in Figure 1.2. Because it costs nothing to enter into a forward contract, the payoff from the contract is also the trader's total gain or loss from the contract.

In the example just considered, $K = 1.2230$ and the corporation has a long contract. When $S_T = 1.3000$, the payoff is \$0.077 per £1; when $S_T = 1.2000$, it is −\$0.023 per £1.

## Forward Prices and Spot Prices

We shall be discussing in some detail the relationship between spot and forward prices in Chapter 5. For a quick preview of why the two are related, consider a stock that pays no dividend and is worth \$60. You can borrow or lend money for 1 year at 5%. What should the 1-year forward price of the stock be?

**Figure 1.2** Payoffs from forward contracts: (a) long position, (b) short position. Delivery price $= K$; price of asset at contract maturity $= S_T$.




The answer is \$60 grossed up at 5% for 1 year, or \$63. If the forward price is more than this, say \$67, you could borrow \$60, buy one share of the stock, and sell it forward for \$67. After paying off the loan, you would net a profit of \$4 in 1 year. If the forward price is less than \$63, say \$58, an investor owning the stock as part of a portfolio would sell the stock for \$60 and enter into a forward contract to buy it back for \$58 in 1 year. The proceeds of investment would be invested at 5% to earn \$3. The investor would end up \$5 better off than if the stock were kept in the portfolio for the year.

## 1.4 FUTURES CONTRACTS

Like a forward contract, a futures contract is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. Unlike forward contracts, futures contracts are normally traded on an exchange. To make trading possible, the exchange specifies certain standardized features of the contract. As the two parties to the contract do not necessarily know each other, the exchange clearing house stands between them as mentioned earlier.

Two large exchanges on which futures contracts are traded are the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CME), which have now merged to form the CME Group. On these and other exchanges throughout the world, a very wide range of commodities and financial assets form the underlying assets in the various contracts. The commodities include pork bellies, live cattle, sugar, wool, lumber, copper, aluminum, gold, and tin. The financial assets include stock indices, currencies, and Treasury bonds. Futures prices are regularly reported in the financial press. Suppose that, on September 1, the December futures price of gold is quoted as \$1,750. This is the price, exclusive of commissions, at which traders can agree to buy or sell gold for December delivery. It is determined in the same way as other prices (i.e., by the laws of supply and demand). If more traders want to go long than to go short, the price goes up; if the reverse is true, then the price goes down.